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George J. Burch

Phil. Trans. R. Soc. Lond. A 1892 **183**, 81-105
doi: 10.1098/rsta.1892.0002

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II. *On the Time-relations of the Excursions of the Capillary Electrometer, with a Description of the Method of Using it for the Investigation of Electrical Changes of Short Duration.*

By George J. Burch, *B.A., Oxon.*

Communicated by Professor Bartholomew Price, *F.R.S.*

Received September 3,—Read November 19, 1891.

[Plates 3–6.]

In a preliminary note "On a Method of Determining the Value of Rapid Variations of a Difference of Potential by Means of the Capillary Electrometer," I indicated briefly the results of an investigation of the time-relations of the excursions of the capillary electrometer. The object of the present paper is to give further details concerning the method of measurement which I then put forward, and to describe the experiments which led to it.

The theory of the instrument has been treated by Lippmann, Hermann, and others, mainly from a mathematical stand-point, and the phenomena have been ascribed to polarization, but no adequate attempt appears to have been made to establish by actual measurement the accuracy of the formula for the time-relations, nor to ascertain whether there were any other influences at work besides those assumed in stating the problem. Poiseuille's experiments on the velocity of the flow of liquids through capillary tubes seem to have been overlooked, and no sufficient investigation made of the effect of the form and dimensions of the tubes upon the rate of movement of the meniscus.

It seemed, therefore, that there was scope for a thorough examination of the problem by the experimental method, with a view, not only to the further elucidation of the theory, but also to the discovery of the best manner of using the instrument so as to render available those special properties which make it unlike any other form of electrometer or galvanometer.

In the course of the investigation, which has occupied more than three years, over a hundred electrometers were made, besides many additional capillaries used in preliminary experiments.

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General Data with respect to the Time-Relations of an Excursion.

(1.) Dilution of the acid beyond a certain point was found to lessen the rapidity of an excursion without altering the extent of it. This indicated that the velocity of the movement was affected by electrical resistance.

A similar result was also produced by the introduction of 100,000 ohms into the external circuit.

(2.) Shortening the capillary so as to reduce the length of the acid column made the instrument act more quickly, but in this case the mechanical friction, as well as the electrical resistance, was lessened.

(3.) The shape of the tube, where it tapers to form the capillary, was found to have a marked effect; the motion of the meniscus being much more sluggish if this part was made too long. In this case the increase of electrical resistance must be comparatively trifling, and the result is to be ascribed mainly to friction and eddy currents.

(4.) The same may be said of the orifice, a clean-cut capillary being usually much quicker in its action than a splintered one.

These things may be taken as indicating the action of both mechanical friction and electrical resistance in determining the rate of movement of the meniscus.

The next step was to ascertain whether there was any latent period before the commencement of the rise or fall. This was done by photography.

A difference of potential was suddenly communicated to the instrument by striking open a short-circuiting key, the end of which projected across the slit upon which the image of the mercury column was thrown, and so gave upon the sensitive plate, as it passed rapidly behind it, a record of the exact instant at which the current was allowed to act upon the electrometer. In no case could the smallest interval be detected between the opening of the key and the commencement of the excursion, even when the difference of potential employed was very small.

The converse of this was found to be true under ordinary circumstances, that is to say, the meniscus ceases to move the moment the source of electromotive force is withdrawn. In other words, the electrometer is practically dead-beat. It has been generally held to be perfectly so, but this is not the case. If the instrument is one that has been specially designed to act with great rapidity, it will be found, on communicating to it a fairly strong charge from a condenser of not more than one-third of the capacity of the electrometer, that the meniscus will start suddenly forward, and *then slowly return*, perhaps as much as one-tenth of the distance it has traversed. But this overshooting is entirely prevented by the introduction of an external resistance of, say, 50,000 ohms. Apparently the reason why this phenomenon has not been noticed is, that experimenters have contented themselves with observing the effect of suddenly breaking the circuit during an excursion; but in so doing, they have introduced an infinite resistance. In the case of the condenser experiment, there

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is, as it were, a reservoir on each side of the electrometer, which acts as a spring, permitting a certain oscillation of the charge. A far more severe test is to preserve the electrometer circuit unbroken, and to remove the source of electromotive force, by breaking the primary circuit of the rheochord. This I have done, and the resulting photographs show the effect of overshooting to be, for the same velocity of the meniscus, nearly twice as great. But it can still be overcome by external resistance, and is only noticeable when the difference of potential is two or three times as great as it ever is in physiological experiments, and even then I have not found it exceed 2 per cent. of the length of the complete excursion, with no external resistance.

It remains true, therefore, that with an ordinarily quick electrometer, under the conditions which obtain in practical work, *the meniscus ceases to move the instant the source of electromotive force is withdrawn.*

The velocity of the movement is also greatest at the first, and there is no portion of the normal photographic curve, even at its commencement, concave to the asymptote, except in those cases in which overshooting may occur. Under such circumstances, slight signs of increase of the velocity are sometimes just discernible. But this was only detected in three cases, when the currents used were as great as the electrometer would bear without injury.

These facts, implying the absence of acquired "momentum," seemed to indicate that in ordinary working the velocity of the meniscus at any moment must be some function of the accelerating force at that moment, in the sense that it is independent of any previous motion, and can contribute nothing—or practically nothing—to the velocity with which it moves during the next interval.

In order to determine experimentally the form of this function, I arranged a rheotome in the derived circuit of a rheochord in such a way that any desired difference of potential could be introduced into the electrometer circuit for an accurately measured period of from ·005 sec. to about ·6 sec. This rheotome, which was upon the principle of a drop-shutter, acted by first snatching open a short-circuiting key, and then after the required interval, breaking the circuit on both sides of the electrometer simultaneously—this arrangement being necessary, in order that the meniscus might stop suddenly without running back to zero. A specially sensitive electrometer was used, having a long range, and with no tendency to "creep." It was less rapid in its action than those usually employed in the laboratory for physiological work, being selected on this account so that the time-measurements might be more accurately made. The insulation was so good that no appreciable return of the meniscus took place during 10 minutes after the circuit was broken.

The experiments were conducted as follows:—Having found the total length of the excursion produced by a difference of potential derived from 150 millims. of the rheochord wire to be 126 divisions of the eye-piece micrometer, the rheotome was set so as to check the meniscus exactly in the middle of the excursion, namely, at 63 divisions from zero. This required a closure of ·414 sec. The rider of the rheochord was then

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shifted to 50 millims., when the complete excursion was found to be 42 divisions, and that given with the rheotome, 21 divisions. On further reducing the length of rheochord wire to 25 millims., the full excursion being 21 divisions, the meniscus shot up, when the rheotome was used, through 10·5 divisions of the eye-piece micrometer and there stopped. That is to say, with only one-sixth part of the original difference of potential, the middle point of the excursion was reached in exactly the same time, namely ·414 sec. In other words, the mean velocity of the meniscus during the first half of a normal excursion was found to be proportional to the total length of it. Further experiments showed that the same was true of other fractions besides the half, as for instance, one-fifth, one-third, and three-fourths, but in these cases the measurements were less reliable, owing to purely instrumental difficulties.

From these data I concluded that the velocity with which the meniscus moved at any instant during a normal excursion, must be proportional to its distance from the final position of rest at that time.

Fig. 1.

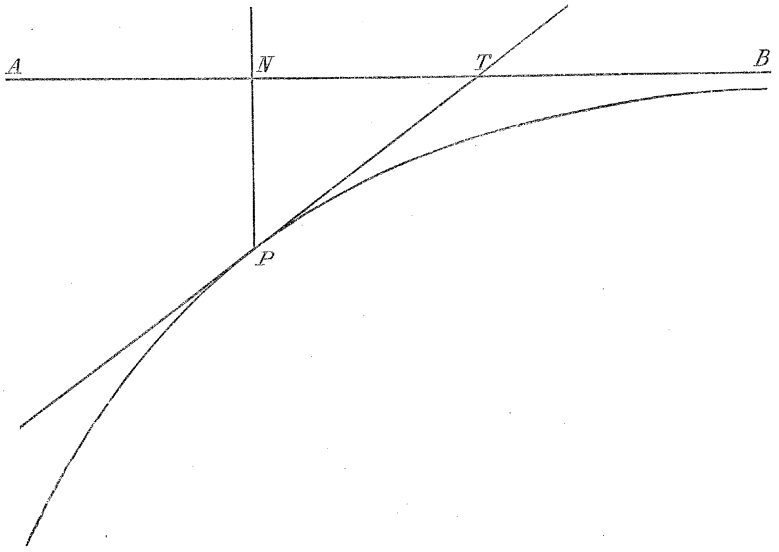

Fig. 1 represents the curve of a normal excursion photographed upon a plate moving with constant velocity horizontally from right to left.

The asymptote AB is taken as the axis along which the time t is measured.

Let P be a point on the curve

$$y = \text{PN} = \text{the vertical ordinate through P.}$$

Then, if u denote the velocity of the meniscus in the capillary

$$u = -\frac{dy}{dt}.$$

But, according to the experimental results,

$$u \text{ varies as } y,$$

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so that

$$cy = -\frac{dy}{dt}, \text{ whence } -c\,dt = \frac{dy}{y}.$$

Integrating

$$\int \frac{dy}{y}\,dt = \log\frac{y}{a} = -ct,$$

or,

$$y = ae^{-ct}.$$

It is obvious that any curve having this equation can easily be recognised in either of two ways. The tabular logarithms of a series of ordinates corresponding to equal time-intervals, are in arithmetical progression. This method was used in verifying the hypothesis as to the time-relations of the normal excursion. The second is a graphic method depending on the fact that the subtangent NT (fig. 1) of a logarithmic curve, or intercept, upon the asymptote between the tangent and ordinate to any point upon the curve, is of constant length. This property affords a means of finding the position of the asymptote when only a portion of the excursion has been included in the photograph. Two points are chosen some distance apart, and the tangent and ordinate drawn to each. The level is then found by trial, at which the horizontal distance between the one tangent and its ordinate is equal to that between the other and its ordinate, or, in other words, the level is found at which the subtangents are constant.

The possibility of doing this in the case of the normal curve suggested the method of analyzing other curves propounded in my preliminary note. If, as my experiments have indicated, the velocity of the meniscus at any moment is due solely to the difference of potential between the terminals of the electrometer at that moment, and is not influenced in any way by its motion at any previous time, and adds nothing to its velocity at any subsequent time—in other words, if the electrometer is perfectly dead-beat—then the rate of movement at any given instant, during an irregular change of electromotive force, must be exactly the same as would be communicated to the meniscus by a permanent difference of potential equal to the electromotive force acting at that instant. That is to say, the tangent to a given point on *the curve produced by any excursion* must coincide in direction with that of the commencement of the *normal curve given by the difference of potential still existing between the terminals of the electrometer.* If, therefore, the length of the constant subtangent to the normal curve is known, the length of the ordinate for that particular inclination of the tangent can be determined, and it will show how far the meniscus has still to move before reaching the position in the capillary corresponding to the difference of potential to which its velocity is due at the time in question. If the mercury is at zero, the length of the ordinate thus determined will represent the actual electromotive force at that instant. If it is not at zero, the distance it has already travelled

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must be added to or subtracted from the ordinate, according as its sign, and that of the ordinate, is positive or negative.

In this way an excursion may be analyzed. Points are taken on the curve at intervals of ·005 or more, and the tangent and ordinate drawn to each and produced, upwards if the curve is rising, and downwards if it is falling, until the horizontal distance between them is equal to the subtangent of the normal curve taken through the same resistance. The ends of all the ordinates are joined, and the curve so produced represents, on the same scale as the ordinary readings of the electrometer, the total difference of potential during each instant of the experiment.

It may be useful to note that the ordinates of the original curve give the integral of the quantity of electricity that has passed at any instant.

Further Investigation of the Formula for the Time-Relations of the Normal Excursion.

Having ascertained, by measuring a number of photographed excursions, that the formula $y = ae^{-\alpha t}$ would hold for some instruments and not for others, I proceeded to investigate the causes of these divergences.

There are two things which may modify the time-relations of the movement—the one accidental, and the other essential. The first is that the instrument may not be of equal sensitiveness throughout the part used. Obviously this difficulty may be got over by selecting a suitable electrometer. The second source of error cannot be so eliminated. It is that the internal resistance varies with the position of the meniscus in the capillary.

(1.) *Calibration Error.*—This has a twofold effect. It alters the electrical value of the scale-readings, and it has also a powerful influence upon the velocity of the movement. The greater the range of the excursion for a given difference of potential, the slower is the action of the instrument—not only relatively, but absolutely. The relation between the sensitiveness of an electrometer, and the time of half-charge, could not be determined directly by experiment, owing to the difficulty of making two capillaries sufficiently alike in all other respects; but from a large number of observations it is evident that the time of half-charge increases much more rapidly than the sensitiveness. I found by experiment that the electrical capacities of electrometers, with capillaries of the same internal diameter, are in direct proportion to the lengths of the excursions produced in them respectively by the same small difference of potential, and that the capacity is within wide limits independent of the difference of potential. As, therefore, the same change of surface-tension has to force the mercury column through distances proportional to the sensitiveness, and a quantity of electricity has to flow which is also proportional to the same thing, it seems probable that the time of half-charge may vary according to the square of the sensitiveness.

Further experiments will be made with regard to this. The important point,

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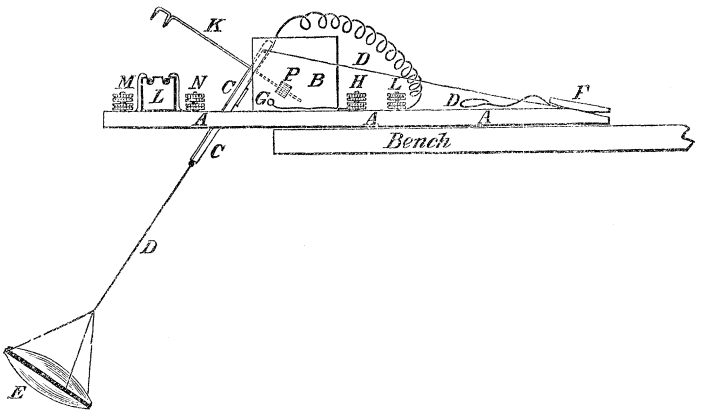
established by a number of observations, is that *the calibration error has a much greater effect upon the rapidity of the movement of the meniscus than upon the scale readings.* In most cases the sensitiveness of an electrometer increases as the meniscus approaches the tip of the capillary, and *its rapidity of action becomes less.* But this calibration error is entirely under the control of the maker of the instrument.

(2.) *Changes of Resistance.*—The principal seat of electrical resistance is in the slender column of dilute acid between the meniscus and the end of the capillary. But since the length of this varies continually during an excursion, it follows that the resistance must also vary, and with it, the velocity of the movement, *which must increase as the meniscus approaches the end of the capillary*, where the resistance is least. This variation was investigated in a series of experiments.

A special electrometer was prepared, with a very long capillary, of exactly equal sensitiveness for a considerable distance from the tip—twice as much as was required for the experiments. It was extremely slow in its action, but was otherwise a very fine instrument.

Fig. 2.



To measure the time-relations of the excursion the following simple and inexpensive rheotome was devised. Upon a base-board *A*, fig. 2, clamped to the bench so as to project beyond it, are fixed two upright pieces *B*, between which the bar *C* swings until it comes in contact, as it assumes a vertical position, with the cross-bar *G*, making connection in so doing between the terminals *H* and *I*, with which it is connected by a fine wire. As it is necessary that the circuit should be broken on both sides of the electrometer, to prevent "creeping," a light wire *K*, balanced by the counterpoise *P*, is attached to *C*, and moves with it. This wire is bent so as to dip into two pools of mercury, in the block of paraffin *L*, thus making a connection between the terminals *M* and *N*, about ·01 sec. before the bar *C* comes in contact with *G*. The weight *E*, which is a disc of lead weighing a couple of pounds, is attached, in the manner shown, to a string *D*, passing through a loop on the end of the bar *C*, over the rod on which it swings, to a cleat *F*. The weight *E* being drawn forward and then let go, carries

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the bar sharply up against the stop G, and then swings under the table, oscillating about the loop at the end of C, as a centre. On its return, as it passes the vertical, the contact is broken, first at G, which determines the period of closure, and immediately after, before the electrometer has time to creep, at L, on the other side of the circuit. It is then caught in the hand of the operator. The closure may be maintained during any odd number of swings by pressing the thumb upon the bar C, directly it has made contact, so as to hold it firmly against the stop, releasing it directly the weight has passed under the table for the last time, thus leaving it free to break contact on its return. The duration of a single swing is calculated by taking the time of 50 complete oscillations with the aid of a stop-watch. By suitably regulating the length of the string D, the time of closure can be adjusted to any desired period from ·4 sec. to 10 secs. The accuracy of the instrument was verified by comparing the results obtained with different lengths of string.

The object of the experiments was three-fold, viz. :—

(A.) To compare the time-relations of charge and discharge.

(B.) To ascertain the difference, if any, of the rate of movement when the meniscus was made to *rise*, *i.e.*, to advance towards the tip of the capillary, and when it was caused to *fall*, *i.e.*, to recede from the tip.

(C.) To measure the effect of the change of internal resistance during an excursion.

As the instrument was one of constant sensitiveness, this could be done by comparing the ratios of successive ordinates corresponding to the equal time-intervals given by 1, 3, 5, 7, etc., swings of the pendulum-rheotome, the mean of a number of observations being taken in each case.

The rheochord was set so as to give an excursion as near as possible of the full length of the scale of the eye-piece micrometer—50 divisions—and the capillary was fixed so that a mark 20 divisions from the tip coincided with the top of the scale. A low power was used, and the actual range of the movement was much greater than is employed in physiological work. One series of observations was taken with an added external resistance of 60,000 ohms. For measuring the rate of discharge, the rheotome was made to close a short circuit between the terminals of the electrometer.

In the following experiments,

l = the distance of the meniscus from the tip of the capillary, in divisions of the eye-piece micrometer.

y = the distance of the meniscus from its final position of rest after a closure of infinite duration.

The period of closure is expressed in swings of the pendulum rheotome.

The last column shows the ratio of each value of y to the one preceding it.

"Up," means an excursion towards the tip of the capillary.

"Down," signifies an excursion away from the tip of the capillary.

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1. CHARGE. No External Resistance. Down.
Period of a single swing of the rheotome = ·36 sec.

Swings.	l.	y.	Ratio.
1	30	40	
3	41	29	1·38
5	48·5	21·5	1·35
7	54	16	1·34

2. CHARGE. No External Resistance. Up.
Period of a single swing of the rheotome = ·36 sec.

Swings.	l.	y.	Ratio.
1	63	44	
3	55	36	1·17
5	49	30	1·20
7	43	24	1·25

3. DISCHARGE. No External Resistance. Down.
Period of a single swing of the rheotome = ·77 sec.

Swings.	l.	y.	Ratio.
1	34·5	35·5	
3	51·5	18·5	1·92
5	60	10	1·85
7	64	6	1·67

4. DISCHARGE. No External Resistance. Up.
Period of a single swing of the rheotome = ·77 sec.

Swings.	l.	y.	Ratio.
1	56	36	
3	45	25	1·44
5	36	16	1·56
7	30	10	1·60
9	26	6	1·67

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5. CHARGE. External Resistance = 60,000 ohms. Up.
Period of a single swing of the rheotome = ·77 sec.

Swings.	l.	y.	Ratio.
1	62·5	42·5	
3	51	31	1·37
5	44·4	22·4	1·38
7	36	16	1·40
9	31	11	1·45

N.B.—In the last three series of observations the period of the swing of the rheotome was made longer, in order to effect a better distribution of the measured points. On plotting the results of identical excursions, the curves were found to coincide with those obtained with a swing of ·36 sec., proving the accuracy of the method to be sufficient for the purpose of these experiments.

The above experiments indicate—

(*a*.) That the time-relations of charge and discharge are practically alike, as is shown also by the analysis of the photographed excursions.

(*b*.) That, in this particular instrument, the meniscus moved more quickly *away* from the tip of the capillary than *towards* it. No such difference could be detected in the case of electrometers of ordinary quickness of action, and the phenomenon appears to be peculiar to those of great sensitiveness and slow action. In some instruments it was reversed, and the mercury would only recede when assisted by tapping the stand, though it would advance freely with a small difference of potential.

(*c*.) That the change of the length of the acid column during an excursion *has* a measurable effect upon the time-relations of the movement, which, however, *is much less than it would be if the normal velocity were conditioned simply by the internal electrical resistance of the electrometer*. It was found by experiments with more than twenty instruments that the effect of an added external resistance in increasing the time of "half-charge" was such, that they might be assumed to have an internal resistance of the form

$$\mathrm{R} = r\,(\mathrm{L} + l),$$

where l = the distance of the meniscus at any moment from the tip of the capillary, and L is a constant of the particular instrument employed. In practice, L is many times greater than the maximum values of l, so that the variation of the speed-ratio from this cause is less than 3 per cent. over the entire width of the negatives referred to in this paper, and consequently not more than $\frac{1}{2}$ per cent. in the physiological curves to which the method of analysis was applied.

To sum up :—

The two causes which modify the time-relations of an excursion, act in opposite ways.

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The decrease of internal electrical resistance as the meniscus approaches the tip of the capillary, tends to increase its velocity, while the increase of sensitiveness makes it move more slowly. But the latter has a much more powerful effect than the former, and may practically neutralize it, or even overpower it, without introducing a calibration error great enough to make an appreciable difference between the scale readings and the true value of an excursion.

The time-relations of the movement are conditioned partly by electrical resistance, but mainly by some other cause—probably mechanical friction.

The Production and Analysis of Photographs of the Excursions of the Capillary Electrometer.

The rapid movement of the sensitive plate required to bring out the details of the electric phenomena of muscle and nerve, necessitated an alteration in the form of the apparatus. The ingenious arrangement devised by the Rev. F. G. SMITH for producing a rectilinear motion of uniform velocity was inapplicable, owing to lack of space in the dark room. I therefore made an apparatus, the details of which will be described at length elsewhere, in which the sensitive plate was caused to describe an arc of a circle. The dark-slide containing it was attached to a kind of balanced pendulum, which carried it, at a uniform velocity, past the slit on which the magnified image of the column of mercury was thrown. The requisite velocity was given to the pendulum by a weight which, as in ATWOOD'S machine, was caught by a stop just before the plate reached the slit. The return of the pendulum after the exposure was prevented by a catch, and the key producing the excursion was actuated at the right moment by an arrangement of electromagnets. Time was recorded upon the plates by a magnetic vibrator placed in front of the slit, and driven by a tuning-fork in the usual way.

The exact instant of excitation was recorded by the signal-key before referred to, which was also placed in front of the slit.

With this arrangement the normal curve is most easily expressed in polar coordinates.

Time being recorded upon a circular arc,

$$t \text{ becomes } \theta.$$

Instead of the rectilinear asymptote, there is an asymptotic circle of radius = R.

The expression for the radius vector is

$$r = \mathrm{R} \pm y,$$

the equation connecting y and θ being

$$y = ae^{-c\theta}.$$

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This change necessitated a reconsideration of the method of analysis. The curve is a variety of the logarithmic spiral, but the intercept between the tangent and the radius vector on the asymptotic circle—the line corresponding to that which I had employed in the case of the rectilinear motion, is not constant, and not of a form convenient to use. There is, however, another property of polar coordinates which is at once available; namely, that the expression for the polar sub-normal is

$$r \cot \psi = \frac{dr}{d\theta}.$$

In this case

$$\frac{dr}{d\theta} = -\frac{dy}{d\theta} = cy,$$

so that this property enabled me to analyze the curves with remarkable ease, and I propose to abandon rectilinear motion for the plates in future.*

Fig. 3.

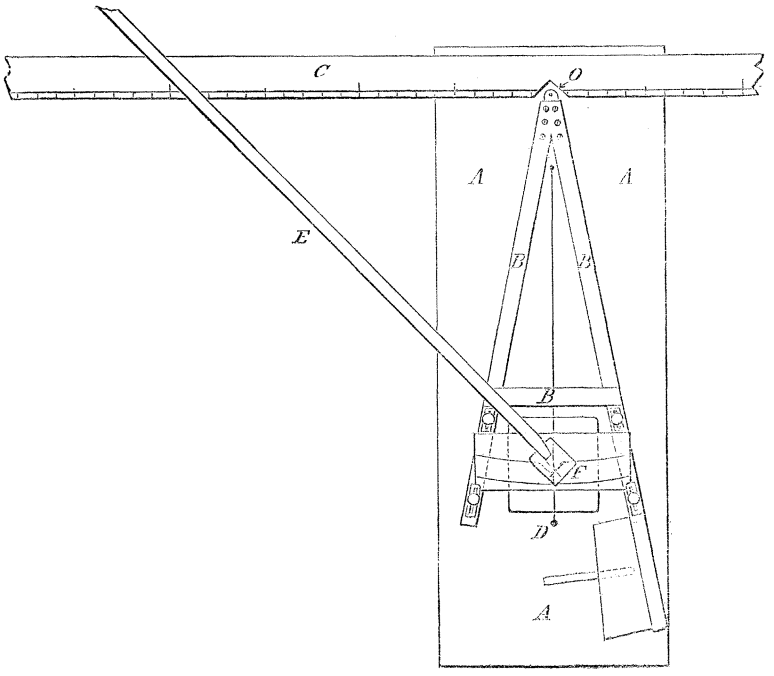
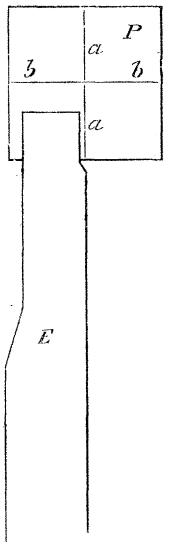


Fig. 4.



The problem of the analysis of a photographed excursion resolves itself therefore into the determination of the length of the subnormals of a sufficient number of points upon the curve, together with the measurement of the distances of these points from the circle corresponding to the position of the meniscus for zero potential. These measurements are most easily made upon the instrument represented in fig. 3.

The negative is fixed to a carrier B, pivoted at O, on the base-board A, in which a

* See WILLIAMSON'S 'Differential Calculus,' 7th edition, p. 223, from which I got the clue while trying to elaborate a roundabout method of reproducing the polar curve in rectangular coordinates, so that I might analyze it.

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hole is cut so that the photograph can be examined by transmitted light. The radius of the carrier is exactly equal to that of the pendulum on which the plate was exposed, and a fine wire or piece of horsehair is stretched from D to O, passing close underneath the glass without touching it, which serves as a radial line of reference. An index fixed to the carrier B, passing over a scale upon A, enables small angular displacements of the carrier to be accurately measured, thus determining the time intervals, and the ordinates are found by laying a graduated rule upon the negative just over the radial line DO, using as a reference circle the edge of the photograph, which is always well defined. The length of the subnormal is found by means of the flat rod, E, which carries at one end the glass plate, F, shown on a larger scale in fig. 4. On this plate is ruled a fine line, a, a_1, continuous with the edge of E, but broken for a distance of about 2 millims. at about the middle. Through the gap thus left passes the tangent line, b, exactly at right angles to a, a_1. When this line b, at the part where it is intersected by a, a_1, is placed as a tangent to the curve which is to be analyzed, at the point where it is cut by the radial line, DO, the length cut off by the rod, E, upon the graduated rule, C, which is permanently fixed at right angles to DO, is the subnormal to the curve at that point. The plate carrier is then shifted through an angle corresponding to a known interval of time, as determined from the time record upon the photograph, and the subnormal measured again for the part of the curve thus brought over the radial line, DO. This process is repeated at sufficiently close intervals throughout the curve, the corresponding ordinates being measured, taking the edge of the photograph as a reference circle.

For the application of this method it is first necessary to analyze a normal curve produced under the exact conditions, as to resistance, of the experimental photographs. With a suitable instrument, the subnormal is practically a constant multiple of y, the distance of the meniscus from its point of rest, and the value of this constant multiplier has to be found.

The following example of the analysis of the normal curve of the electrometer used in the physiological experiments described by Professor Burdon Sanderson in a paper on the "Photographic Determination of the Time-relations of the Changes which take place in Muscle during the period of so-called Latent Stimulation," ('Proceedings,' vol. 48, p. 14), will serve to show how this is done. In this particular case, as the curve was obtained for the purpose of analyzing the physiological photographs which we had taken, the circuit was led through the non-polarizable electrodes and the muscle exactly as it had been arranged for those experiments. An equivalent metallic resistance was then substituted for the preparation, and several other photographs of the normal excursion were secured. But the resulting curves were found to give identical results and, therefore, the details of this one only are inserted here.

First, the ordinates corresponding to time-intervals of ·001 sec. were measured with an ivory rule, graduated very finely in millimetres, the tenth of a millimetre being

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estimated with the aid of a lens. As a check, three different reference lines were taken on successive days, and the results so obtained were found to be identical.

The next stage of the operation consisted in the determination of the asymptotic circle. In so short a period as that comprised between the passage across the slit of an eight-inch plate, the mercury has not time to reach its final position, which must therefore be found by calculation. Assuming for the time-relations of the excursion the simplest formula, given in my preliminary note, viz.:—

$$y = ae^{-ct},$$

we may write for the distances from any reference circle of three points corresponding to equal intervals of time,

$$\mathrm{A} = n + y,$$

$$\mathrm{B} = n + \mathrm{T}y,$$

$$\mathrm{C} = n + \mathrm{T}^2y,$$

where n = the distance of the reference circle from the asymptotic circle, and T is some function of t.

Whence

$$\frac{\mathrm{C} - \mathrm{B}}{\mathrm{B} - \mathrm{A}} = \mathrm{T};$$

$$\frac{\mathrm{B} - \mathrm{A}}{\mathrm{T} - 1} = y;$$

$$\mathrm{A} - y = n.$$

Selecting three points as far apart as possible, I found the position of the asymptotic circle and reduced the observed readings of the ordinates to the values of y measured from it, which were as follows:—

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ANALYSIS of Normal Curve No. 289, fig. 5, Plate 4. Upward excursion equivalent to the difference of potential given by 83 centims. of the rheochord wire with 1 Callaud cell. Resistance in circuit, a gastrocnemius preparation with non-polarizable electrodes. Total length of the excursion = 31·7 millims.

Found.	Calculated.	Difference.
millims.	millims.	millim.
*y = 28·000	28·800	
24·900	24·855	+ ·045
21·400	21·451	− ·051
18·500	18·513	− ·013
16·000	15·978	+ ·022
13·800	13·789	+ ·011
11·900	11·901	− ·001
10·200	10·271	− ·071
8·900	8·864	+ ·036
7·650	7·650	

The column of calculated values is a series in geometrical progression, inserted between the first and the last terms. It will be seen that the error is in all cases well within the limit of accuracy of the measurements.

Having thus determined the position of the asymptotic circle, *i.e.*, the position of the meniscus when $y = 0$, on the assumption that the formula $y = \alpha e^{-\sigma t}$ was exact, the next step was to try whether any traces could be discovered of the influence of change of resistance, or of the sensitiveness of the capillary, during the excursion. In other words, I had to determine whether these errors were due to inaccuracies of measurement or of the formula assigned to the curve. If the latter were the case, then there must be some value of n which would make T increase or decrease with some approach to regularity from one end of the curve to the other, and we should have approximately

$$\log (n + y_1) - \log (n + y_2) = \log \mathrm{T},$$
$$\log (n + y_2) - \log (n + y_3) = \log (\mathrm{T} + b),$$
$$\log (n + y_3) - \log (n + y_4) = \log (\mathrm{T} + 2b),$$

and so on. On assuming trial values for n, however, the discrepancies seemed only to increase, and, after a somewhat laborious investigation, I concluded that in this particular instrument the effect of diminished resistance was practically balanced by a slight increase of sensitiveness towards the tip of the capillary—the one tending to increase, and the other to diminish, the velocity of the meniscus, so that the equation $y = \alpha e^{-\sigma t}$ represents very closely the time-relations of the excursion. The process

* These ten points correspond to intervals of ·01 sec., as given by the time-tracing. That is why the first one is some 3 millims. from the commencement of the curve.

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above described is necessary with every new electrometer, in order to determine its constants. When the equation of the curve has thus been determined once for all the process is much more simple. The position of the asymptotic circle is found by direct measurement upon a screen placed in the position of the sensitive plate, and a single normal excursion of known value is taken through the resistance used in the experiments to which the method of analysis is to be applied. Two or three ordinates with their corresponding subnormals are then measured, and the ratio of each ordinate to its subnormal is determined.

The mean of these ratios is taken as the value of the constant multiplier.

Measurement of the Subnormal of Normal Curve No. 289.

The curve was spoilt for this purpose by the presence of small undulations in two places caused by some jar to the apparatus; these only interfered with the measurement of y at one point, where, however, it was easy to take the mean position between them. But in order accurately to place the fine line upon the glass plate of the measuring rod as a tangent to the curve, it was necessary that this should be perfect for some little distance on each side of the point of contact. The two most favourable positions were near the beginning and the end of the curve, and these gave for the value of C respectively

$$C = 8{\cdot}50,$$

$$C = 8{\cdot}51.$$

Two other positions, not so well situated, gave $C = 8{\cdot}40$ and $C = 8{\cdot}43$. The remaining readings were less reliable on account of the undulations referred to, and a short piece where the definition of the photograph was defective. The mean of the four measurements is $C = 8{\cdot}46$; but $8{\cdot}50$ is probably nearer the true reading.

With this instrument, therefore, a difference of potential due to 83 centims. of the rheochord wire gave an excursion of 31·7 millims. on the sensitive plate. With a resistance in circuit equal to that of an ordinary physiological preparation, the subnormal to the curve at its commencement was 26·945 centims.

Whence

$$1 \text{ centim. on the subnormal} = \frac{83}{26{\cdot}945} \text{ centims.} = 30{\cdot}8035 \text{ on the rheochord,}$$

and

$$1 \text{ centim. on the rheochord} = \frac{26{\cdot}945}{83} \text{ centims.} = {\cdot}32464 \text{ on the subnormal.}$$

Comparison with other Normal Excursions.

In order to show that the method may be relied on to give constant results, the following experiments are quoted. The circuit was the same as for Curve No. 289,

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and the value of each excursion, as measured by its subnormal, may be compared with that obtained by calculation.

No. 290.—Rider of the rheochord set at 67 centims. Muscle-current = 42 centims., giving an effective difference of potential of 25 centims. Subnormal at the commencement of the excursion = 8·19 centims. Calculated

$$83 : 25 :: 26{\cdot}945 : 8{\cdot}12.$$

No. 292.—The difference of potential was simply that due to the muscle-current, which had fallen to 37 centims. The excursion was therefore in the opposite direction. Subnormal at the commencement of the excursion = 11·955 centims. Calculated

$$83 : 37 :: 26{\cdot}945 : 12{\cdot}01.$$

These results agree fairly well. For greater accuracy, the actual measurement of the subnormal was made in all cases at a point from 2 millims. to 5 millims. from the commencement, and the difference of the ordinates of the measured point and of the zero line, multiplied by the constant factor 8·5, added to the result. Hence the number of places of decimals. (See Part II., p. 101.)

The muscle was then removed from the circuit, and the following experiments made.

No. 294.—Upward excursion (towards the tip of the capillary) given by 80 centims. of the rheochord wire. No external resistance. Subnormal at the commencement of the curve = 28·2 centims.

No. 295.—Upward excursion given by 80 centims. of the rheochord wire. External resistance = 1000 ohms. Subnormal at the commencement of the curve = 28·04 centims.

No. 296. Downward excursion given by 80 centims. of the rheochord wire. External resistance = 1000 ohms. Subnormal at the commencement of the curve = 28·14 centims.

These results show the identical character of the upward and downward excursions, and also that the introduction into the circuit of 1000 ohms makes no very marked difference; they do, however, indicate the effect of taking away the resistance of the muscle and its electrodes, in that the value of the constant multiplier has risen from 8·5 to about 9·2.

Artificial Spikes.

This name was given to the curves obtained by photographing the excursions produced by means of two currents in opposite directions, each lasting about ·005 sec. It was intended to imitate as nearly as possible the effect produced by the excitation of muscle, with differences of potential of known value and duration, so as to determine whether the capillary electrometer was capable of distinguishing between a current of definite strength suddenly thrown into it, and a more or less gradual rise of a difference of potential, extending over a period of equal duration. That it can do so was shown beyond a doubt. To produce these artificial spikes, the wires from the electrometer were joined up through the derived circuits of two rheochords, A and B, each of which was provided with a separate battery. These were connected

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with a rheotome furnished with four adjustable break-keys of the tip-over type. The first of these, when struck, opened a short circuit on the primary of A, allowing the current to act upon that rheochord. The second broke the battery circuit of A, and at the same instant the third in like manner opened a short circuit on the primary of the second rheochord B, in which the current circulated in the opposite direction. About ·005 sec. later the battery of B was thrown out of action by the fourth key, thus checking the excursion produced by the second current. It will be observed that the electrometer circuit remained unbroken throughout the experiment.

Unfortunately the short-circuit contacts were not good, so that a certain amount of current passed, rendering it impossible to ascertain the exact value of the differences of potential which acted upon the electrometer, in several cases—and, moreover, the Daniell's cells used were not sufficiently steady during the operations, which took a long time, to enable all the results to be compared with those previously obtained. The experiments were varied by reversing the direction of both currents, and sometimes omitting the second.

No. 306. Fig. 6, Plate 3.—"Artificial Spike," produced by two short currents of opposite direction. Up, ·0055 sec.; subnormal = 50 centims. Down, ·0045 sec.; subnormal = 60 centims.

It will be observed that the effect of the second current was to bring the meniscus back almost exactly to its former level, and that both the rise and fall commenced and ended suddenly, indicating, as do also the measurements of the subnormal at close intervals throughout the curve, that it was produced by the introduction of differences of potential which remained constant while they lasted. But the movement was in each case only about one-tenth of the full excursion for that difference of potential, and the quantity of electricity received as "charge," viewing the electrometer as a condenser, is approximately proportionate to the product of the time into the difference of potential, or what comes to the same thing, the subnormal, *i.e.*:—

$$\cdot0055 \times 50 = \cdot275 \text{ for the upward movement.}$$
$$\cdot0045 \times 60 = \cdot270 \text{ for the downward movement.}$$

But these "charges" may be measured in another way. The electrical capacity of an electrometer is perfectly definite in amount, being conditioned by the shape and size of the capillary, and perfectly independent of the difference of potential. The quantity received by an instrument from any current is therefore proportional to the product of its capacity (which is constant throughout the part of the capillary used) into the distance l, through which the meniscus is moved by it. The effect of two successive charges of equal quantity, but opposite in sign, must be to leave the mercury ultimately at zero, as was the case here. Not only was there this close correspondence between the values obtained in these two ways for the "charge," but the Total Indicated Differences of Potential, calculated from the subnormals, were within one per cent. of the values found by comparing the two rheochords employed, with that used for Curve No. 289.

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The following curves are given as examples of the effects of overshooting. The actual difference of potential employed was not ascertained owing to the instrumental difficulties referred to, which were not noticed till the conclusion of the experiments.

No. 305. Fig. 7, Plate 3.—Artificial spike, given by an upward excursion lasting 0·0052 sec., followed by a downward excursion lasting 0·004 sec.

The second phase is slightly rounded at the beginning, showing that the full velocity was not attained immediately. The slight pause at the end of the fall, before the slow return to zero, is probably due to "stickiness" of the tube. (Compare also No. 299 where there is evidence of "stickiness" during the swiftest part of the movement).

No. 307. Fig. 8, Plate 3.—Artificial spike, as before, but in the reverse direction.

In this case, the second phase begins sharply at its full velocity, but at the end of it the mercury has continued to move, not so rapidly as when the current was on, but still, more quickly than it would have done had the electrometer been simply discharging itself.

No. 309. Fig. 9, Plate 3.—Excursion produced by a single current lasting 0·006 sec.

Here the meniscus overshot the mark to the extent of 0·7 millim. in an excursion, the full height of which would have measured 80 millims. on the photograph, and which had only risen 7 millims. when the current was checked. The analysis shows that the second rheochord somehow became connected and furnished a current in the opposite direction of about one-fourth of the difference of potential of the first—possibly one of the leads might have touched the rheotome.

It will be noticed that even this overshooting is under 1 per cent. of the total normal excursion, and that the time which elapsed between the withdrawal of the electromotive force and the cessation of the movement, was only about 0·002 sec.

These three photographs illustrate the form of overshooting, which I consider to be due to the elasticity of the meniscus, for, if it were true overshooting of the column, then the retardation of the return movement should be visible in the case of the reversed spike (No. 307, fig. 8); but this was only seen with much stronger currents. On the other hand, the sudden checking of an upward excursion, about three times the size of this, caused the meniscus to throw off a globule of mercury.

In the preceding experiments, there was no external resistance. The effect of introducing a coil of 10,000 ohms is shown in the next example.

No. 299. Fig. 10, Plate 3.—Downward excursion produced by a single current of short duration. External resistance = 10,000 ohms.

In spite of the difference of potential being greater than in any of the preceding experiments, there is practically no overshooting, and the curve of discharge is perfectly normal. The subnormal to the curve was too great to be measured on the

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machine. The slight notch about the middle of the descent, indicates a "sticky" place in the capillary. (Compare fig. 7.)*

A companion photograph, in which the excursion was upward, did not develop sufficiently dense to print, but was similar in character, the upward overshooting being very slight.

These photographs show, that *without external resistance in the circuit*, the effect of overshooting, and especially of that form of it which is due to the elasticity of the meniscus, interferes seriously with the application of this method of analysis, unless the differences of potential concerned are small, but that *these difficulties vanish on the introduction of a moderate resistance*, not sufficient to make the movements of the electrometer sluggish. With suitable precautions, the variations of a difference of potential may be determined for each two-thousandth part of a second, and under favourable circumstances, with an error of not more than one per cent.

Part II.

Application of the Method to the Study of the Electrical Variations of Muscle.

In a paper by Professor Burdon Sanderson, entitled "Photographic Determination of the Time-relations of the Changes which take place in Muscle during the Period of so-called 'Latent Stimulation'" ('Roy. Soc. Proc.,' vol. 48, p. 14), it was stated that in the gastrocnemius of the Frog the electrical response to an instantaneous stimulus, as investigated with the aid of the capillary electrometer, is indicated by a sudden movement of the mercurial column of very short duration, and that the photographic expression of that movement shows that between the contacts two electrical changes of opposite sign, and not more than one two-hundredth of a second in duration, have immediately followed each other, or, more explicitly, that the spot excited became for about 0·005 sec. first negative, then for a similar period positive, to the other contact.

This statement relates exclusively to the case in which the effect is led off to the electrometer by two electrodes of which one (f) is placed on the tendon, and the other (m) is on the belly of the muscle. In this case, the photographic record of the electrical response to a single excitation of the nerve, resembles the record obtained, when in a circuit of the same resistance as the muscle, two currents of the same duration follow one another in opposite directions.

This resemblance between the two cases is, however, only general, for whereas in the artificial effect the difference of potential between the two terminals of the electrometer remains the same during each phase, changing sign at the moment that the first is followed by the second, the difference of potential between the two spots to

* [This notch has not been reproduced in the figure. It can easily be seen in the negative even without a lens. March 3, 1892. G. J. B.]

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which the electrodes are applied gradually rises to a negative maximum during the first phase, is reversed, and then, having become positive, gradually subsides.

My present purpose is to show in what way the method of analysis described can be applied to the photographic records of the electrical response (which I shall hereafter speak of as the record or curve), so as to determine the exact time-relations of the changes above described.

With this view I propose to present to the Society certain specimen records, and to state the results obtained by the analysis of each. This result will be best expressed by a graphic denoting the actual differences of potential between (m) and (f) at successive times during the period of electrical change.

It may be convenient at this point to describe in detail the method of analyzing such records as muscle curves, in which the electrical variations are of brief duration and not necessarily of constant intensity or of the same sign.

It has been shown in the first part of this paper that it is not difficult to procure an electrometer of which the time-relations may be expressed by the formula $y = ae^{-\alpha t}$, and that, with a circuit of suitable resistance, the movement of the meniscus will commence the instant that a difference of potential is communicated to the terminals, and cease the instant it is withdrawn. With such an instrument under such conditions, as was stated in my preliminary note "On a Method of Determining the Value of Rapid Variations of a Difference of Potential by means of the Capillary Electrometer" ('Roy. Soc. Proc.,' vol. 48, p. 92), "the velocity with which the meniscus is moving at any instant is that with which it would start if the zero-line were moved to the level then occupied by the meniscus, and the difference of potential existing at the time between the terminals of the electrometer were suddenly introduced and made permanent. Thus, the *total indicated difference of potential* is made up of two parts, viz., the difference represented by the distance through which the meniscus has already moved, and that indicated by the velocity with which it is still moving."

In the analysis of muscle curves, as will be seen, the latter is usually many times greater than the former, except at the beginning, middle, and end of the curve where the tangent is at right angles to the radius vector, and the subnormal is zero.

The method of analysis set forth in my preliminary note is, in consequence of the change in the apparatus already referred to, whereby the sensitive plate is made to describe an arc of the circle instead of travelling in a straight line, superseded by the far simpler one given in the present paper. The subnormal to the normal curve—and, consequently, to any curve—is a constant multiple of y, the distance of the meniscus from the point of rest corresponding to the difference of potential existing at the time between the terminals of the electrometer.

As has been already said, the value of this constant multiplier is influenced by the resistance of the circuit in each experiment, and the first step is to determine it for the particular resistance used. This was done for the muscle curves of which the

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analysis is appended, the normal excursion photographed for the purpose being No. 289 (fig. 5), and the value of the constant multiplier was 8·5.

The next step is to place the glass negative of the muscle-curve upon the measuring instrument (fig. 3), carefully centreing it so as to ensure its occupying a position exactly corresponding to that in which it was placed on the pendulum of the photographic apparatus. If this is not done, the results will be falsified. The rate of movement has then to be determined. When, as in the cases given here, the angle subtended by the entire excursion is small, no appreciable error is introduced by employing a tangent scale instead of measuring the arc. This being so, a very simple method is available, whereby the hundredths of a second, given by the time-record on the photograph, can be subdivided into thousandths of a second. The index of the carrier (fig. 3), is a straight edge fixed radially, and the tangent scale, which is divided into millimetres, is placed at such a distance from the centre O, that ten of its divisions shall be passed over by the index when the carrier is shifted through a space corresponding to one-hundredth of a second, as indicated by the coincidence of two successive apices of the time-record on the photograph with the radial line DO.

The scale having been fixed in this position, the actual measurements are proceeded with. The instant of excitation, marked by the upward movement of the signal arm, is taken as the zero of time. This portion of the photograph is brought over the line DO, and the reading of the tangent scale is noted. The distance of the electrometer record at that time from the circular edge of the photograph is measured; this gives the position of zero potential. The next thing is to find the time at which the meniscus began to move. For this purpose, the rod E is laid along the line DO, and the line aa, on the glass plate F, placed accurately as a tangent to the curve. The carrier is then shifted until a point is reached when the curve commences to rise, and the time at which this takes place is noted on the scale.

Next, the carrier is further shifted in the same direction through a space corresponding to 0·0005 sec.

There are now two operations to be performed:—

(a.) To measure the subnormal of the curve at this point. The method of doing this has been already described. The result may be written

$$\text{Subnormal} = cy.$$

(b.) To measure how far the meniscus has moved from its original zero position for $t = 0$.

This is done by taking the distance of the curve at this point from the edge of the photograph, which is used as a reference circle, and subtracting the result from the zero distance already ascertained. The difference may be written

$$\text{Movement} = l.$$

It is most convenient to express the total indicated difference of potential in terms

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of the subnormal—reducing it subsequently to decimals of a volt. The total indicated difference of potential will therefore be

$$\pm cy \pm cl.$$

Attention must be paid to the *sign of both y and l*. In the examples given, an upward excursion is produced by the negative variation, so that the formula for the first phase will be

$$-cy - 8{\cdot}5l.$$

At the apex the subnormal passes through zero, but the total indicated potential is not yet zero, being indicated truly by the distance of the meniscus from the zero line, *i.e.*, the actual height of the excursion. This, expressed on the same scale as before, is

$$-8{\cdot}5l.$$

During the second phase, or descending portion of the curve, the subnormal is positive, and we must write

$$+cy - 8{\cdot}5l.$$

If, as sometimes happens, the meniscus should descend below the zero line, l also becomes positive, and we have

$$+cy + 8{\cdot}5l.$$

It is unnecessary to describe minutely the arrangement of the physiological preparations. The photographic curves of which the analysis is given are some of those obtained in the experiments in which I assisted Professor Burdon Sanderson, to whose kindness I am indebted for permission to make use of them in this, the first application of the method of analysis which this paper describes.

Non-polarizable electrodes were employed, one (f) placed on the tendon, and the other (m) on the belly of the muscle, unless otherwise stated. The muscle was kept stretched by a weight, not greater than could be lifted by a moderate contraction. The exciting electrodes were applied to the nerve, the exact instant of the stimulus being marked by the upward movement of the arm of the signal-key, the shadow of which is recorded on the plate. This key, which has been already referred to, was placed in the primary circuit of the induction-coil, and by its use, all possibility of delay in registering the instant of the induction shock was eliminated, save that due to the induction coil itself.

No. 287. Fig. 11, *a*, Plate 4.—Gastrocnemius of *R. esculenta*. Exciting electrodes placed at the extreme end of the prepared nerve, as far as possible from the muscle.

The muscle current was balanced by the difference of potential derived from 42 centims. of the wire of the rheochord. The analysis of this curve is given in fig. 11. The excursion is not large, but the negative variation is sudden, and remains tolerably constant for some time. The positive variation is not great, but the return to the original zero is slow.

The relative value of the muscle current is shown by the dotted line, which represents the true zero potential.

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Immediately after this experiment, without altering the electrometer circuit, the normal excursion, No. 289, was photographed for the determination of the constant multiplier used in this and the remaining illustrations.

No. 270. Fig. 12, *a*, Plate 5.—Gastrocnemius. Exciting electrodes as far as possible from the muscle.

No. 271. Fig. 12, *b*.—The same muscle. Exciting electrodes as close as possible to the muscle.

No. 272. Fig. 12, *c*.—The same muscle. Exciting electrodes as close as possible to the muscle as in No. 271, but the leading-off electrode (*m*) shifted from its previous position to a point 12 millims. nearer the tendon electrode (*f*).

These three curves were analyzed, and the results plotted together in fig. 12, Plate 5.

It will be observed that the two former, though somewhat different in the first phase, are fairly alike in the second, the delay of the electrical response, in consequence of the greater length of nerve through which the excitation was transmitted in the case of No. 270, being well shown.

It is interesting to note that the analysis of No. 272 approaches more nearly in shape to that of an "artificial spike." The rise of the electromotive force is fairly rapid, and it remains nearly constant for a considerable part of each phase. But neither the rise nor the fall is so sudden as in the case of Curve No. 306, although, as the difference of potential is less and the resistance of the circuit greater, any error due to overshooting of either kind is quite out of the question.

No. 274. Fig. 13, *a*.—Gastrocnemius. Exciting electrodes placed on the nerve, close to its insertion in the muscle. Leading-off electrode (*f*) attached to the tendon; electrode (*m*) on the side of the muscle, nearly at its thickest part.

No. 276. Fig. 13, *b*.—The same muscle. Exciting electrodes as before. Leading-off electrode (*m*) shifted into the normal position on the belly of the muscle.

No. 277. Fig. 13, *c*.—The same muscle. Exciting electrodes as before. Leading-off electrode (*m*) placed so close to the tendon electrode (*f*) as to give a scarcely visible excursion of the meniscus upon excitation.

The analyses of these three curves were plotted together in fig. 13, Plate 6.

They all cross the zero line within a remarkably small space. The first two differ slightly in the negative phase, but are tolerably similar in the positive, which is well developed. In both there is a second rise of the positive variation, reaching a maximum about 0·023 sec. after the excitation. This rise is, however, too small to be discernible from the photograph, and is made manifest only by the method of analysis.

It does not come within my province to discuss the physiological questions arising out of these results, which I desire to leave entirely in the hands of Professor Burdon Sanderson, to whom I am deeply indebted for permission to make use of these photographs, taken in the course of the research in which I had the honour of assisting him, in order to illustrate the method of analysis which I now bring forward.

My sole object has been to show that the capillary electrometer can be employed to obtain measurements which hitherto could only be effected by a process open to some objections.

The repeating rheotome, introduced by Professor Bernstein for the same purpose,

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although it has many advantages, can only give the average time-relations of a number of electrical variations due to a rapid succession of stimuli, and is inapplicable in any case in which changes may occur in the preparation from fatigue. Moreover, my experience of rheotomes leads me to believe that it is much more difficult than is sometimes supposed to obtain very short closures of constant duration. The photographic records given by the capillary electrometer are free from both these objections. They show the result of a single stimulus or of a number in rapid succession, and the circuit remains unbroken and therefore constant.

Hitherto no means by which these indications could be quantitatively interpreted, except in the case of slow changes, has been in the hands of experimenters. I have endeavoured to supply this lack, and in doing so, to represent the capillary electrometer in its true light as a measuring instrument, not indeed equal to the galvanometer in delicacy, nor in durability, nor, perhaps, in accuracy; but far excelling it and all other electrical measuring instruments in the one property which is peculiar to it, namely, rapidity of action.

Probably in this way the electrometer may be made to serve other purposes than those of physiological research; and I foresee that this method of analysis may be applicable to other instruments than the electrometer.

All the photographs and many of the experiments described in this paper were made in the Physiological Laboratory, Oxford, and my thanks are due to Professor Burdon Sanderson for his help and counsel, without which I could not have completed the research. I have also to thank Mr. C. Crump, of the Record Office, for his valuable assistance in discussing the mathematical parts of the problem. The complete method of applying the corrections for calibration error and change of resistance I had hoped to give with his aid, but it was found that some further experiments would be necessary to furnish the requisite data. The treatment of this part of the subject therefore must be deferred. In the instrument which was used for nearly all the photographs these errors were not sensible.

[Note.—The curves were analysed by measuring the negatives, and the results plotted from left to right. The photographs having been reproduced as prints, must be read in the reverse direction, from right to left. March 3, 1892. G. J. B.]

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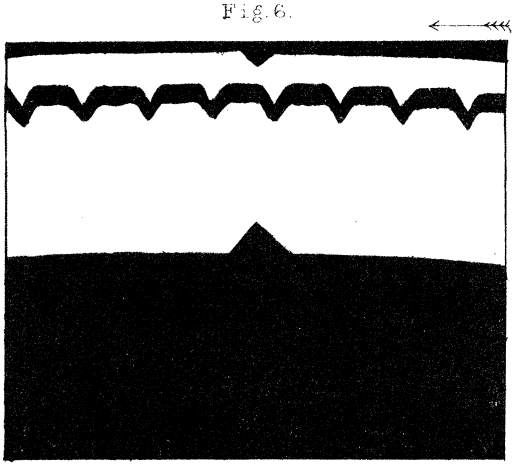
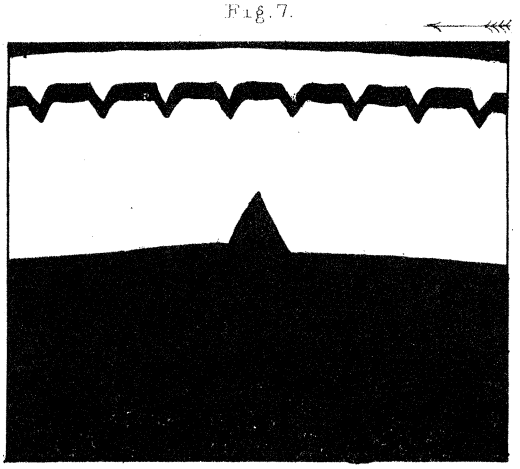
Fig. 6.



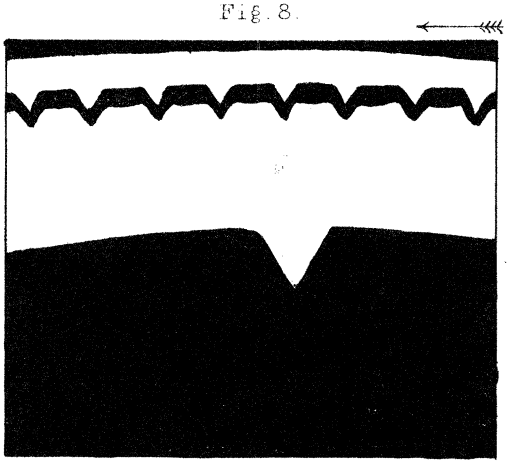
No. 306.

Fig. 7.



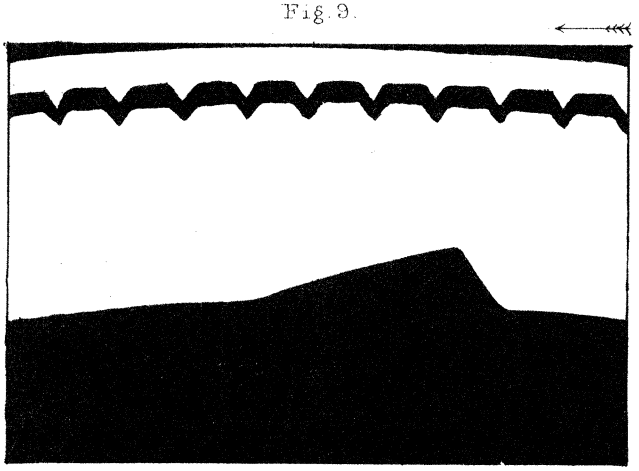
No. 305.

Fig. 8.



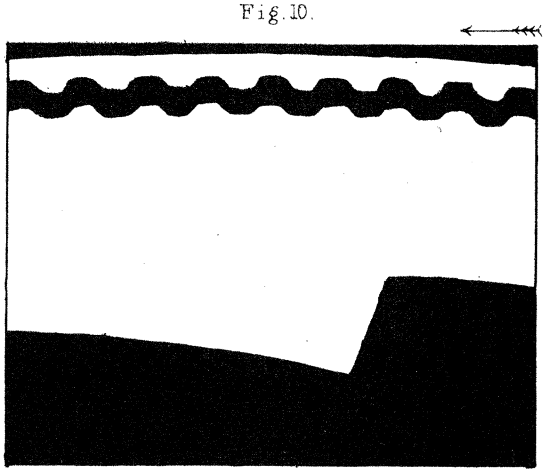
No. 307

Fig. 9.



No. 309.

Fig. 10.



No. 299.

West, Newman lith.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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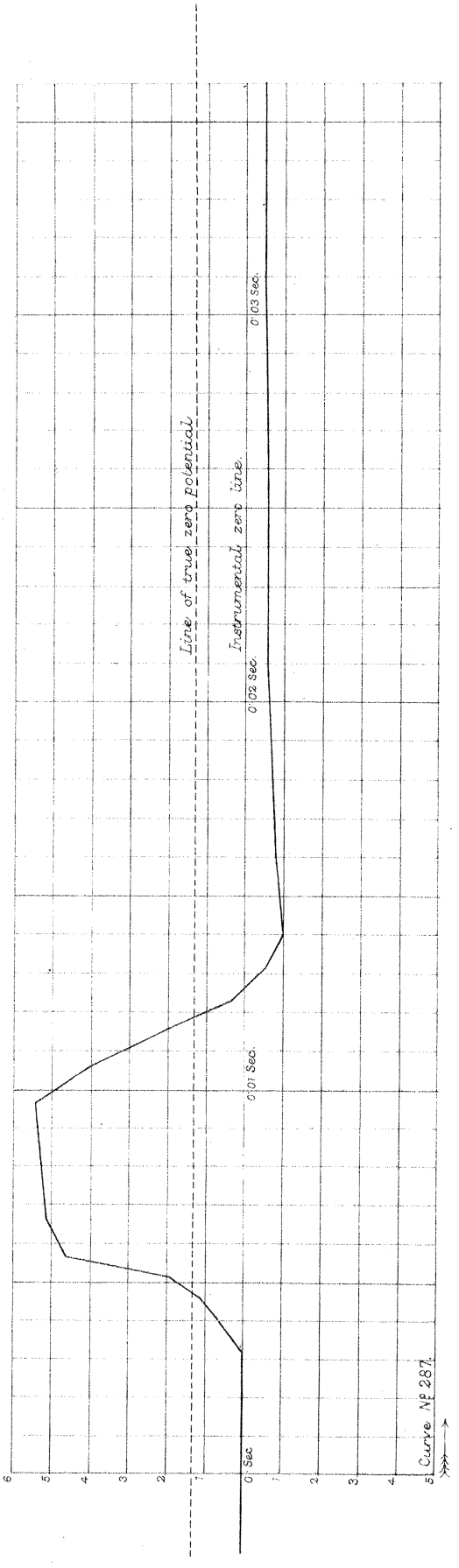


Fig. 11.

(Vertical scale reduced to ½ that of Fig. 11a.)

N.B. Fig. 11, must be read from left to right, but Figs. 5 and 11a, from right to left.

Fig. 11a.

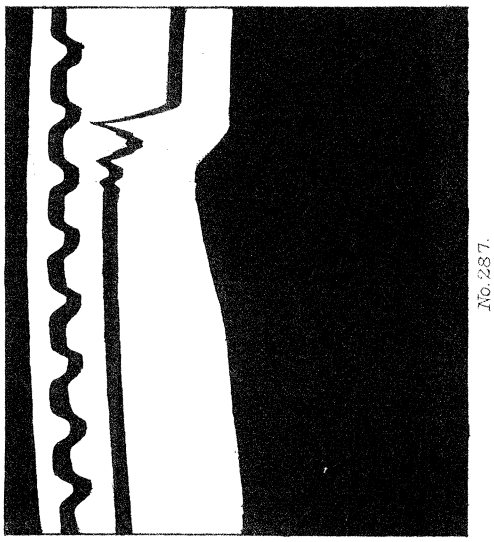
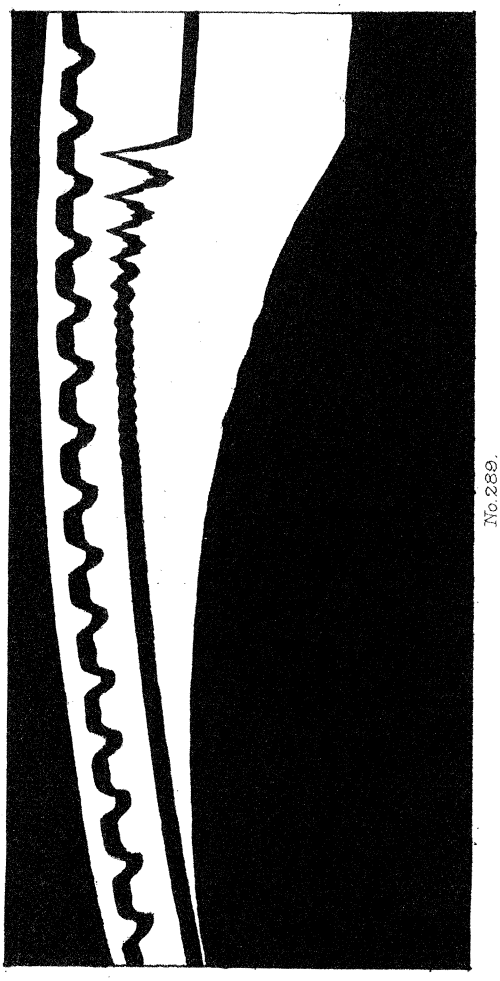


Fig. 5.



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PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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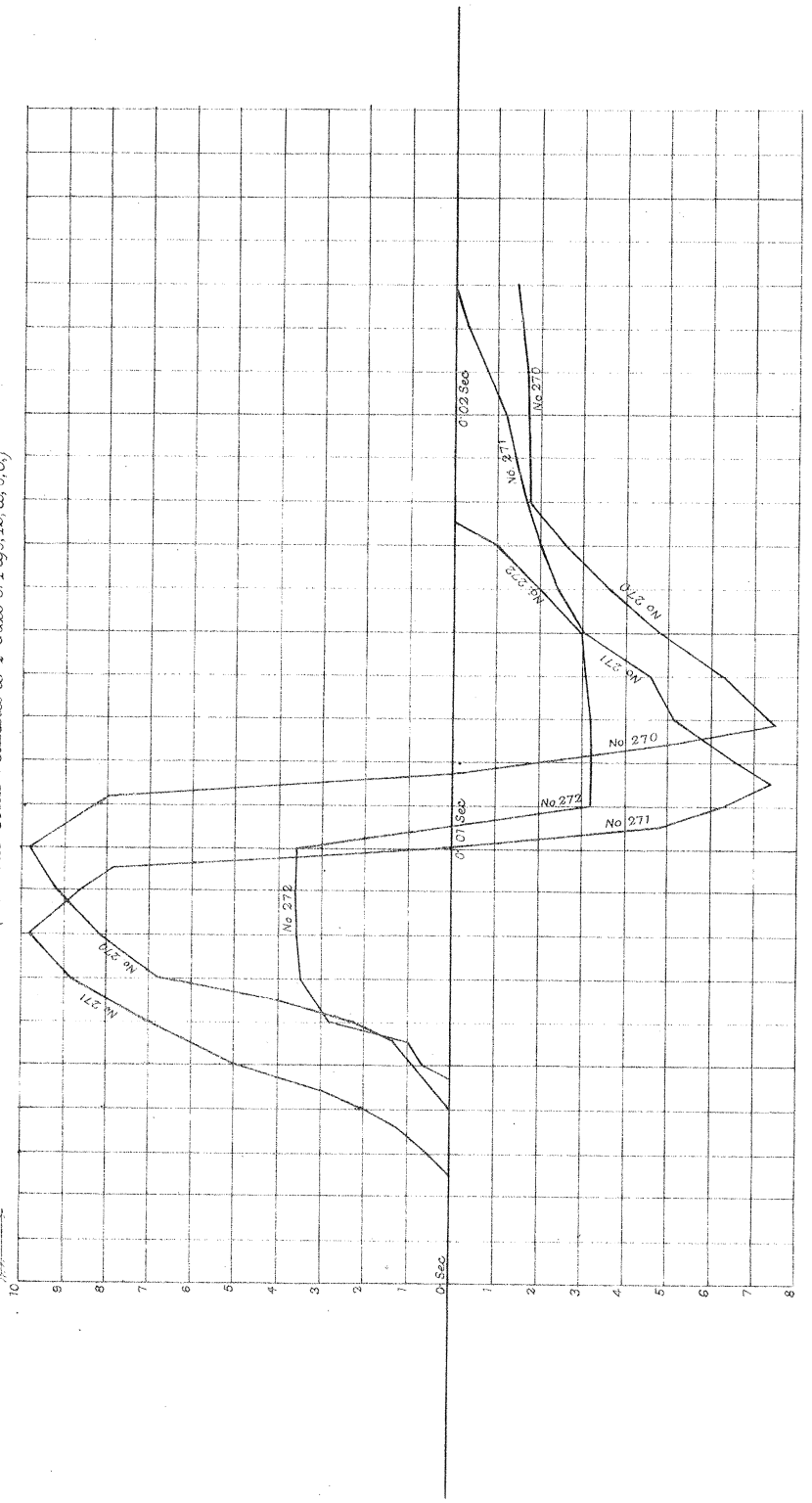

(Vertical scale reduced to ½ that of Figs. 12, a, b, c.)

N.B. Fig. 12 must be read from left to right, but Figs. 12, a, b, c, from right to left.

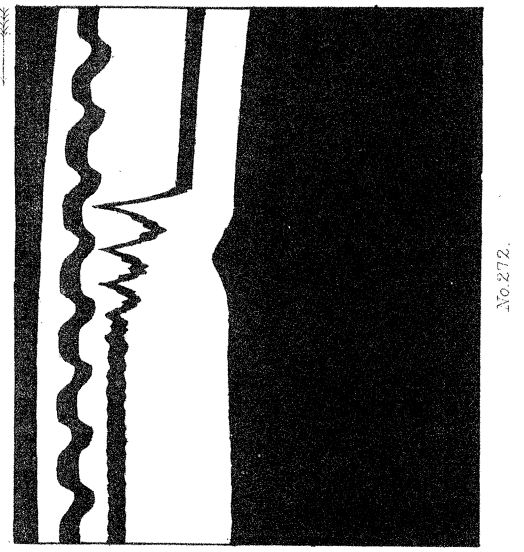

Fig. 12. c.

No. 272.

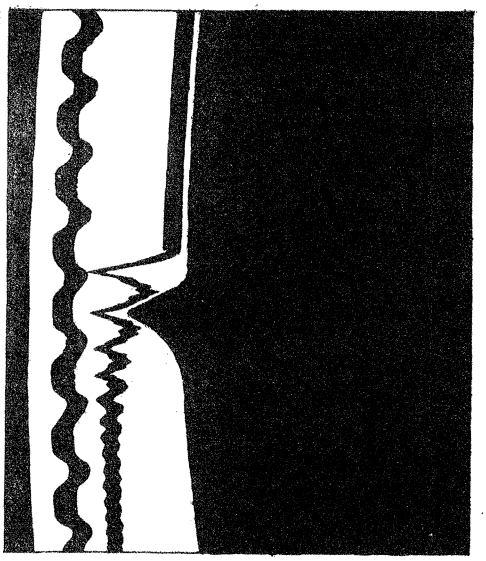

Fig. 12. b.

No. 271.

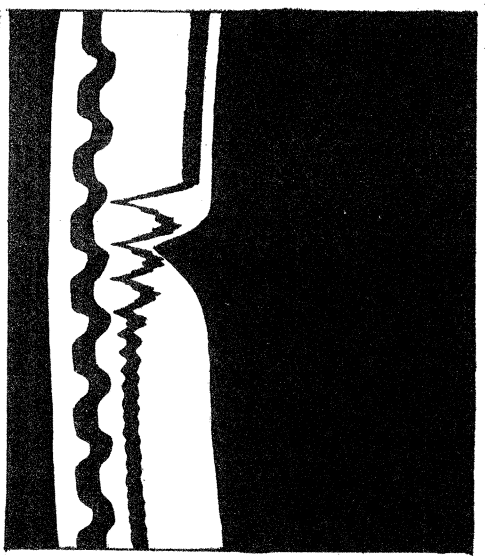

Fig. 12. a.

No. 270.

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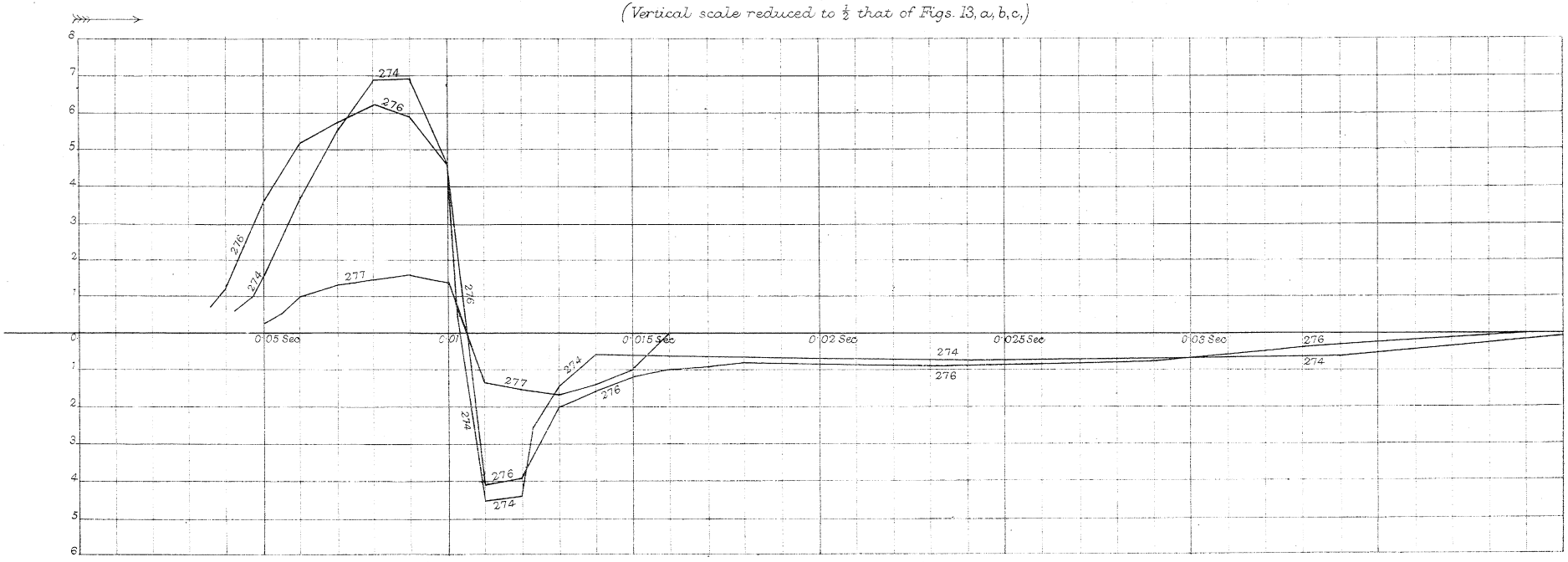
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

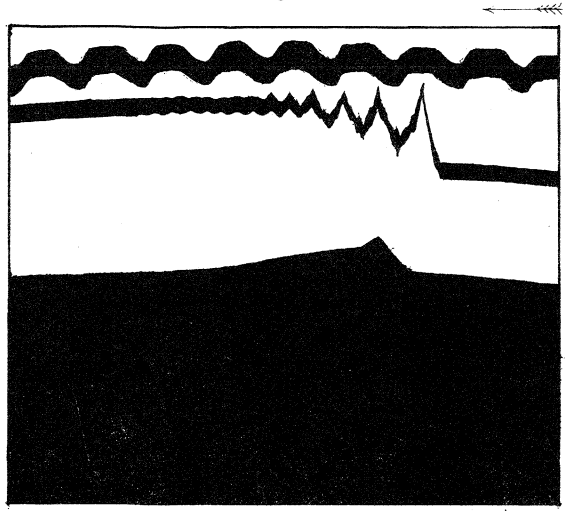
Fig. 13.

(Vertical scale reduced to ½ that of Figs. 13, a, b, c.)



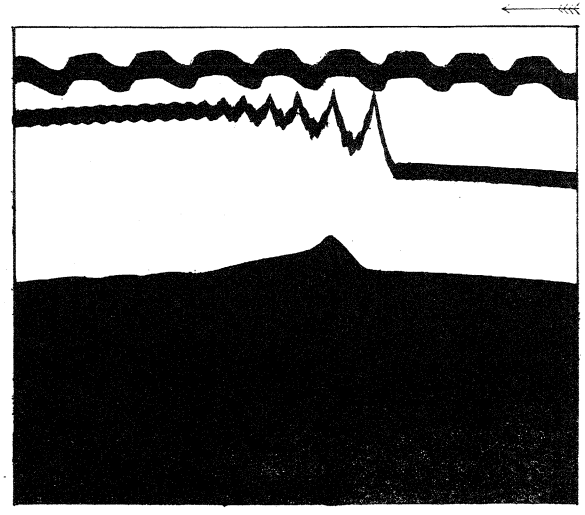
N.B. Fig. 13 must be read from left to right, but Figs. 13, a, b, c, from right to left.

Fig. 13 *a*.



No. 274.

Fig. 13 *b*.



No. 276.

Fig. 13 *c*.

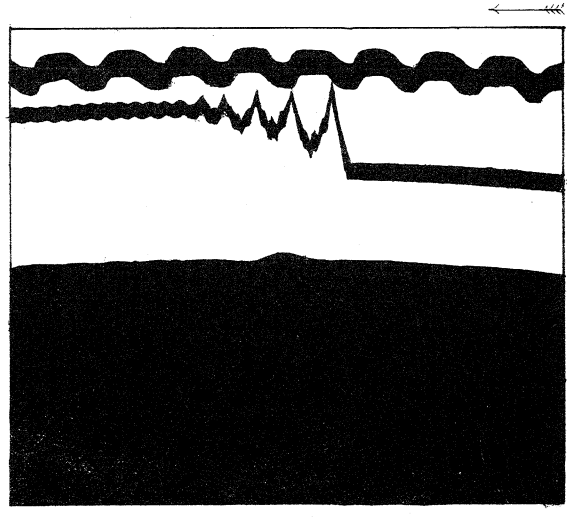


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Fig. 6.

No. 306.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 7

No. 305.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE SOC

Fig. 8.

No 307

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE SOC

Fig. 9.

No. 309.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY

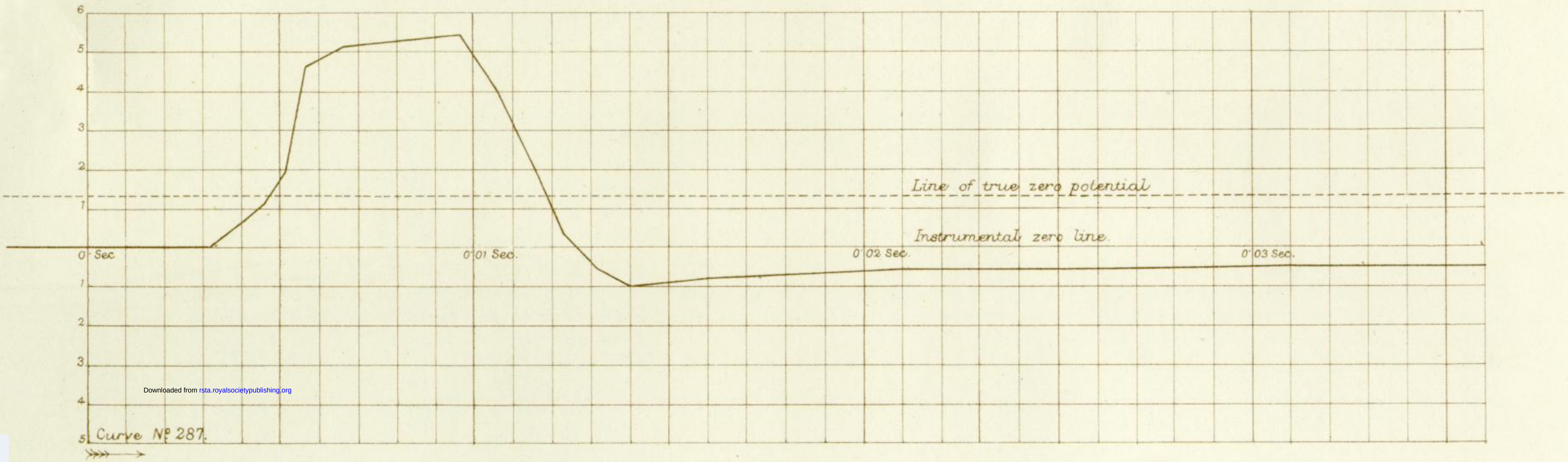
Fig. 10.

No 299.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 11.

(Vertical scale reduced to ½ that of Fig. 11, a.)



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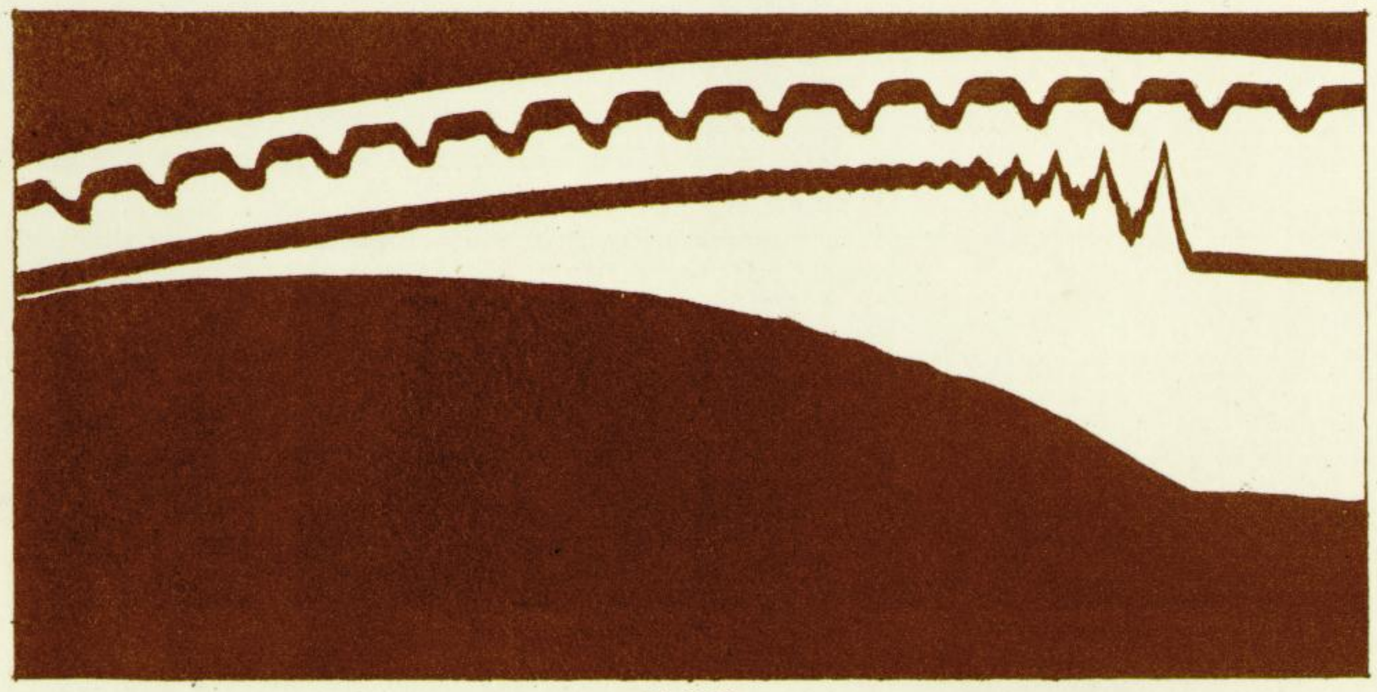
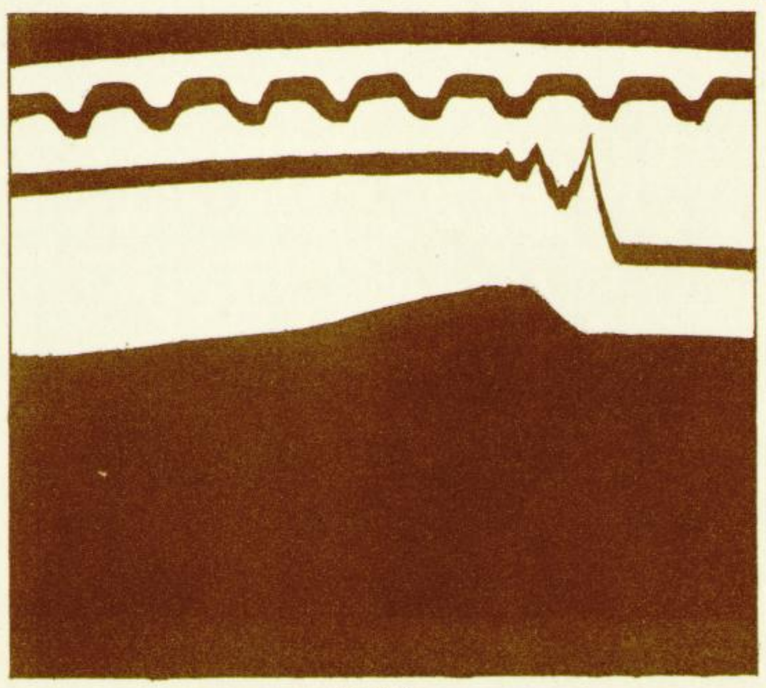
N.B. Fig. 11, must be read from left to right, but Figs. 5 and 11, a, from right to left

Fig. 5.



No. 289.

Fig. 11a.

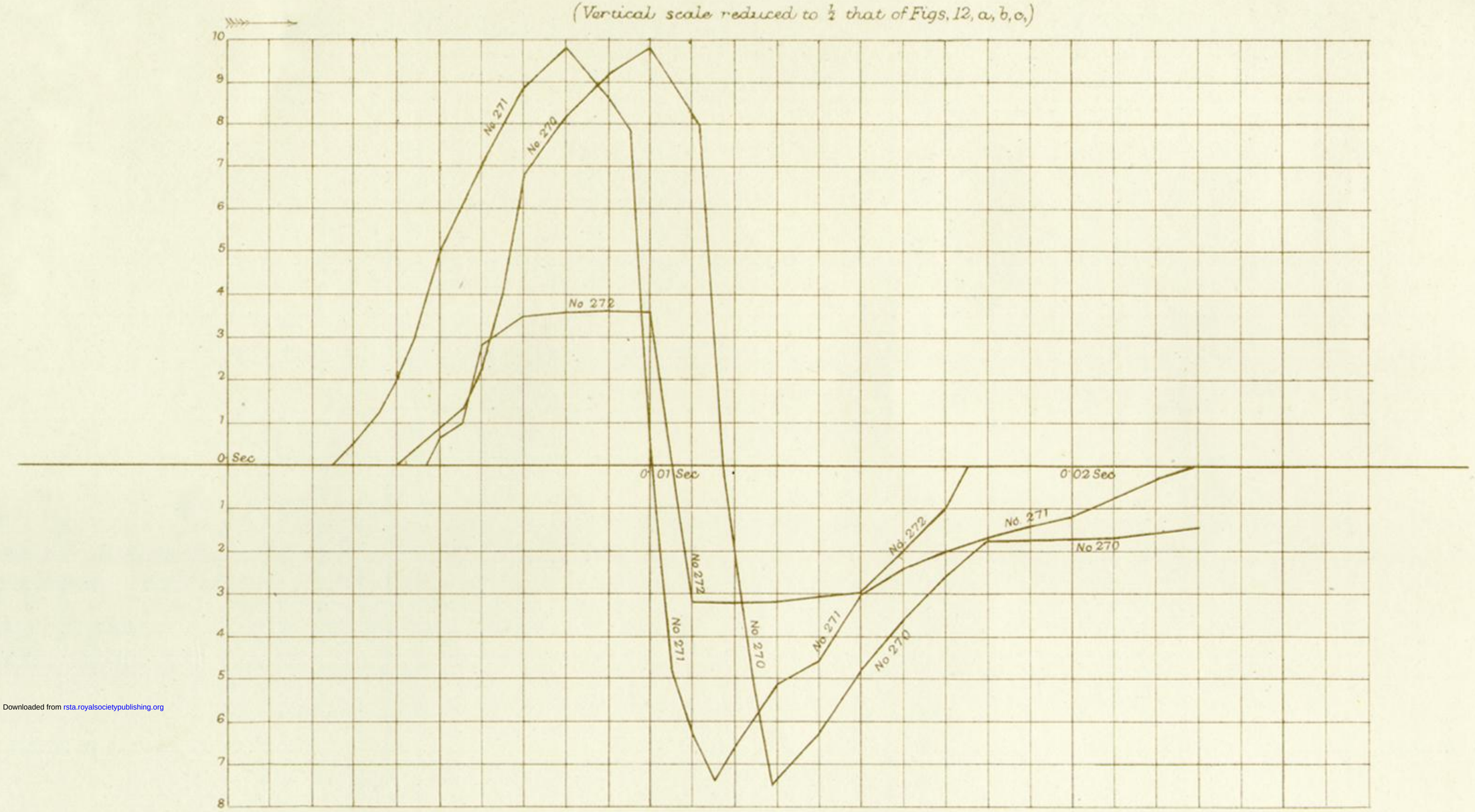


No. 287.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 12.

(Vertical scale reduced to ½ that of Figs. 12, a, b, c,)



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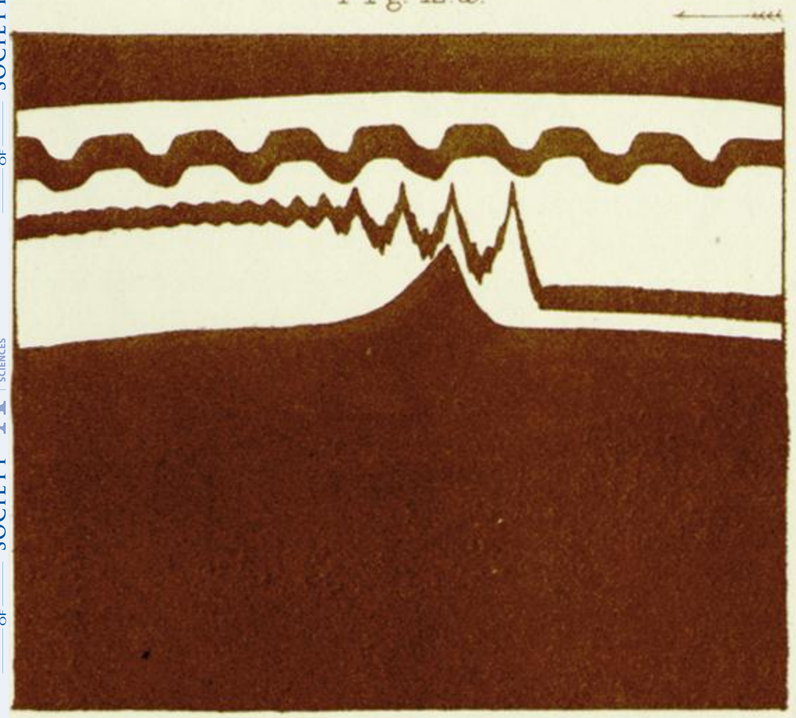
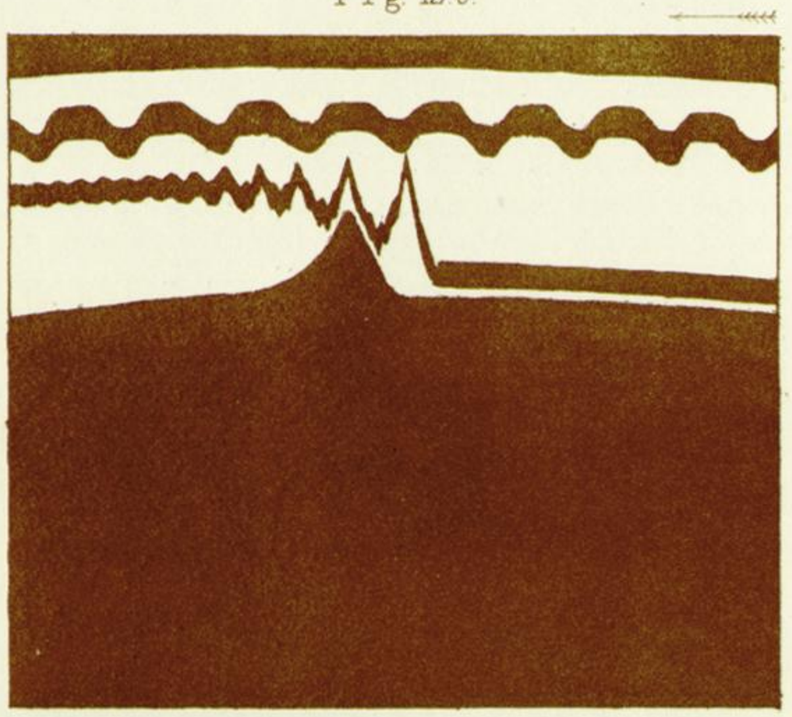
N.B. Fig. 12 must be read from left to right, but Figs. 12, a, b, c, from right to left.

Fig. 12.a.



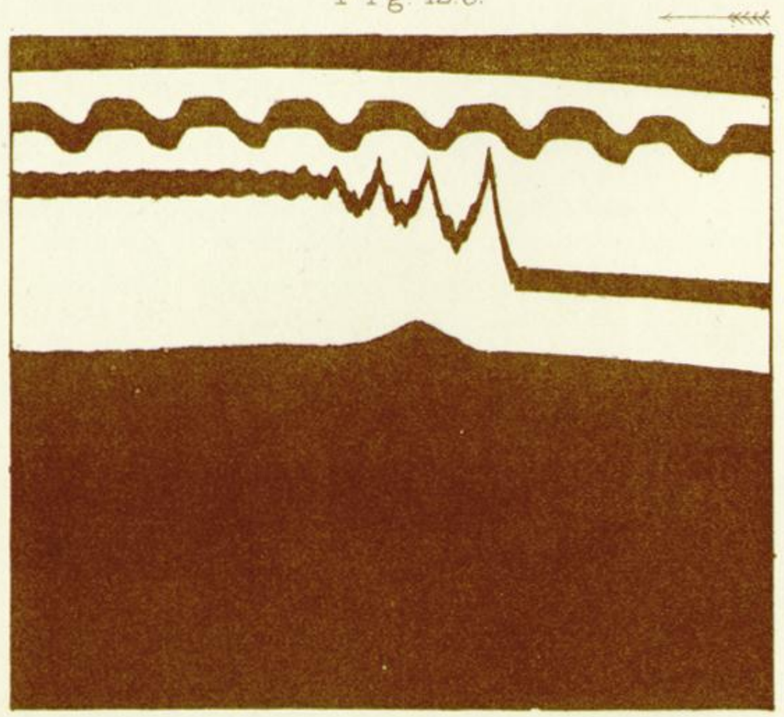
No. 270.

Fig. 12.b.



No. 271.

Fig. 12.c.



No. 272.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 13.

(Vertical scale reduced to ½ that of Figs. 13, a, b, c,)

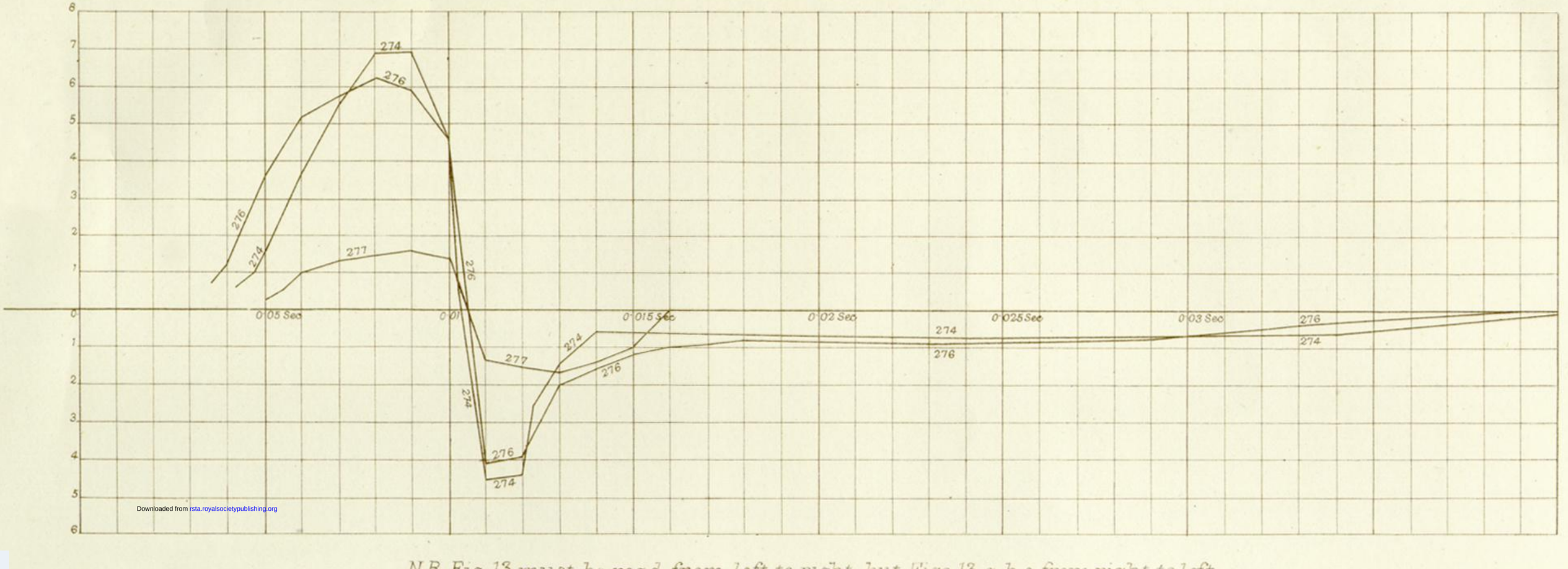


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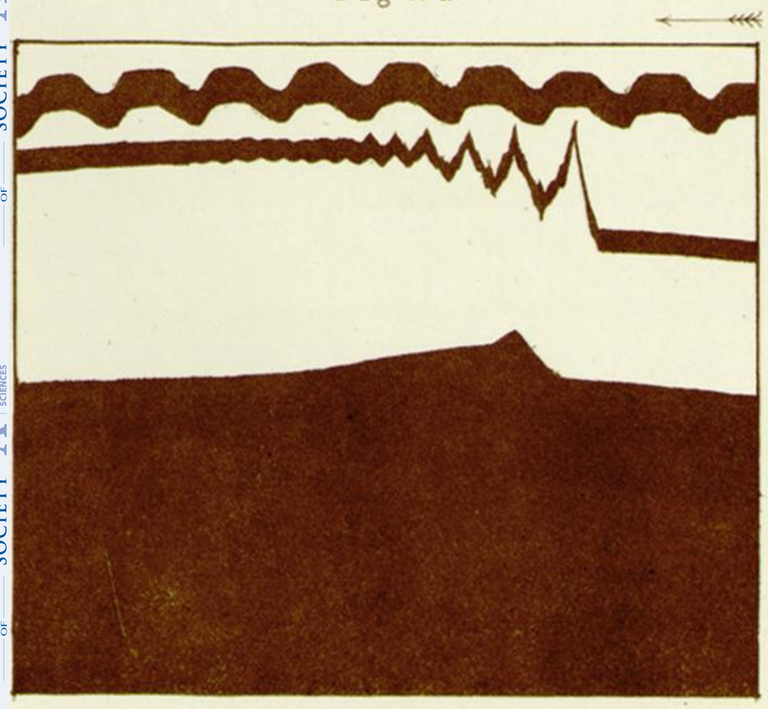
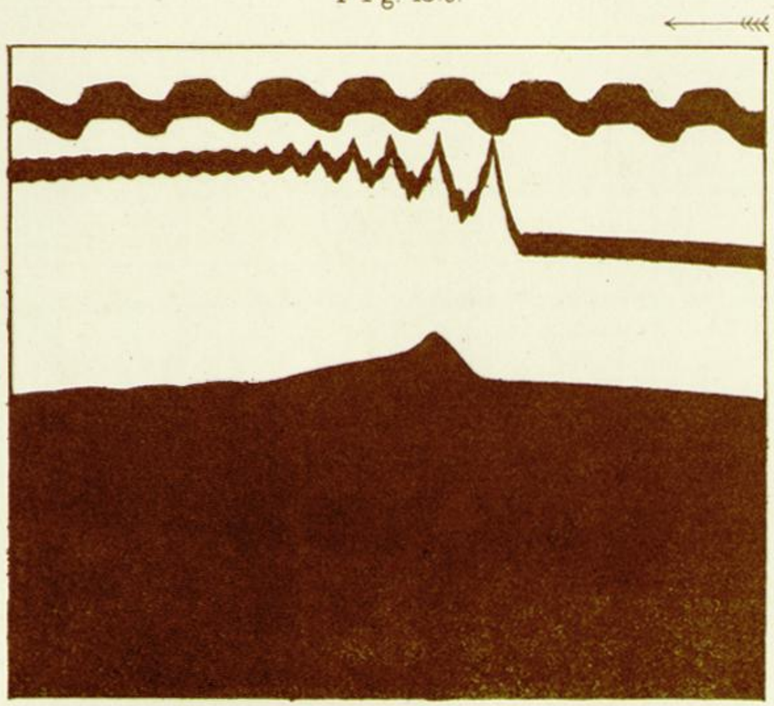
Fig. 13.a.



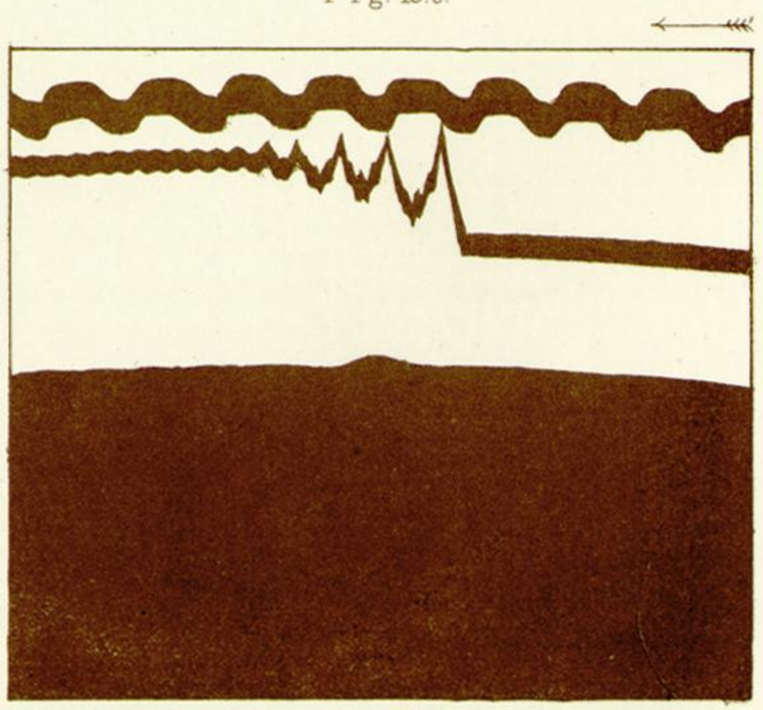
No. 274.

Fig. 13.b.



No. 276.

Fig. 13.c.



No. 277

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES